

Charged-lepton-flavour violation in kaon decays in supersymmetric theories^{*}

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Abstract. We discuss rare kaon decays that violate charged-lepton flavour conservation in supersymmetric theories with and without R parity, in view of possible experiments using an intense proton source as envisaged for a neutrino factory. In the minimal supersymmetric standard model, such decays are generated by box diagrams involving charginos and neutralinos, but the limits from $\mu \rightarrow e\gamma$, μ - e conversion and Δm_K constrain the branching ratios to challengingly small values. However, this is no longer the case in R -violating theories, where such decays may occur at tree level at rates close to the present experimental limits. Within this framework, we obtain bounds on products of $LL\bar{E}$ and $LQ\bar{D}$ operators from the experimental upper limits on $K^0 \rightarrow \mu^\pm e^\mp$ and $K^{\pm,0} \rightarrow \pi^{\pm,0} \mu^\pm e^\mp$ decays. We also note the possibility of like-sign lepton decays $K^\pm \rightarrow \pi^\mp \ell^\pm \ell^\pm$ in the presence of non-zero \tilde{b}_L - \tilde{b}_R mixing. We conclude that rare kaon decays violating charged-lepton flavour conservation could be an interesting signature of R violation.

1 Introduction

The recent Super-Kamiokande data [1] have triggered an upsurge of interest in extensions of the standard model with massive neutrinos and/or violation of the charged-lepton numbers in processes such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\tau \rightarrow \mu\gamma$ and $\mu \rightarrow e$ conversion on heavy nuclei [2–6]. The present experimental upper bounds on the most interesting of these processes are

$$\begin{aligned} \text{BR}(\mu \rightarrow e\gamma) &< 1.2 \times 10^{-11} \quad [7], \\ \text{BR}(\mu^+ \rightarrow e^+ e^+ e^-) &< 1.0 \times 10^{-12} \quad [8], \\ \text{BR}(\mu^- T i \rightarrow e^- T i) &< 6.1 \times 10^{-13} \quad [9], \\ \text{BR}(\tau \rightarrow \mu\gamma) &< 1.1 \times 10^{-6} \quad [10]. \end{aligned} \quad (1)$$

Our main interest in this paper is in rare kaon decays, for which the current experimental bounds are

$$\begin{aligned} \text{BR}(K_L^0 \rightarrow \mu^\pm e^\mp) &< 4.7 \times 10^{-12} \quad [11], \\ \text{BR}(K_L^0 \rightarrow e^+ e^-) &= (8.7_{-4.1}^{+5.7}) \times 10^{-12} \quad [12], \\ \text{BR}(K_L^0 \rightarrow \mu^+ \mu^-) &= (7.18 \pm 0.17) \times 10^{-9} \quad [13], \end{aligned}$$

$$\begin{aligned} \text{BR}(K^+ \rightarrow \pi^+ \mu^+ e^-) &< 2.8 \times 10^{-11} \quad [14], \\ \text{BR}(K^+ \rightarrow \pi^+ \mu^- e^+) &< 5.2 \times 10^{-10} \quad [15], \\ \text{BR}(K_L^0 \rightarrow \pi^0 \mu^\pm e^\mp) &< 3.1 \times 10^{-9} \quad [16], \\ \text{BR}(K^+ \rightarrow \pi^+ e^+ e^-) &< (2.94 \pm 0.05 \pm 0.14) \times 10^{-7} \quad [17], \\ \text{BR}(K_L^0 \rightarrow \pi^0 e^+ e^-) &< 4.3 \times 10^{-9} \quad [18], \\ \text{BR}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) &< (7.6 \pm 2.1) \times 10^{-8} \quad [19], \\ \text{BR}(K_L^0 \rightarrow \pi^0 \mu^+ \mu^-) &< 5.1 \times 10^{-9} \quad [20], \\ \text{BR}(K^+ \rightarrow \pi^- \mu^+ e^+) &< 5.0 \times 10^{-10} \quad [15], \\ \text{BR}(K^+ \rightarrow \pi^- e^+ e^+) &< 6.4 \times 10^{-10} \quad [15], \\ \text{BR}(K^+ \rightarrow \pi^- \mu^+ \mu^+) &< 3.0 \times 10^{-9} \quad [15]. \end{aligned} \quad (2)$$

Projects are being proposed that could be used to improve these limits significantly, e.g., some of the powerful proton sources being proposed for neutrino factories [21] could provide intense secondary kaon beams.

Any observable rate for one of these processes would constitute unambiguous evidence for new physics. The rates for such processes remain extremely suppressed if we simply extend the standard model to include right-handed neutrinos, but larger rates are possible in more ambitious extensions of the standard model. Supersymmetry is one example of new physics that could amplify rates for some of the rare processes (1) and (2), either in the minimal su-

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persymmetric extension of the standard model (MSSM) or in its modification to include violation of R parity.

In previous works, rare charged-lepton decays and anomalous $\mu \rightarrow e$ conversions on heavy nuclei have received considerable attention [6], leading to a good understanding of the correlations between the predicted rates and possible violations of leptonic universality and/or exotic Yukawa couplings with $\Delta L \neq 0$. It is natural to include rare kaon decays in this analysis, because strangeness-changing decays occur in the standard model. As mentioned above, there is now considerable discussion of intense proton sources to be used as muon sources for neutrino factories [21]. If the protons have high energy above $\simeq 15$ GeV, as in some neutrino factory designs, they would also be copious sources of kaons. These might provide a new opportunity to study rare K decays with high statistics, and we are interested to know whether these might cast additional light on neutrino masses and mixing. Specifically, as we show in this paper, rare K decays that violate charged-lepton number allow one, in the context of the MSSM, to correlate the (s)quark and (s)lepton mixing, whilst in R -violating supersymmetry one can probe interesting products of Yukawa couplings.

This paper is organized as follows. In Sect. 2, we analyze the decays $K^0 \rightarrow \mu^\pm e^\mp$ in the MSSM, finding rates that are relatively small, although for certain model parameters they may be within the reach of an imaginable kaon beam at a neutrino factory. In Sect. 3, we analyze the same decays in R -violating supersymmetric models, finding that the rates could in principle be much larger, close to the present experimental limits. Indeed, these existing upper limits on these decays already impose interesting upper limits on products of LLE and $LQ\bar{D}$ couplings. We also point out that \tilde{b}_L - \tilde{b}_R mixing in the presence of R violation could lead to the like-sign-lepton decays $K^\pm \rightarrow \pi^\mp \ell^\pm \ell^\pm$, although the existing experimental limits on these processes do not impose interesting upper limits on couplings. Taken together, our results indicate that rare K decays could provide interesting signatures for supersymmetric models, in particular those with R violation.

2 Rare kaon decays in the MSSM

In this section, we evaluate the rates for rare kaon decays in the MSSM with massive neutrinos, using the see-saw mechanism [22], which we consider to be the most natural way to obtain neutrino masses in the sub-eV range. In particular, we assume Dirac neutrino masses m_ν^D of the same order as the charged-lepton and quark masses, and heavy Majorana masses M_{ν_R} , leading to a light effective neutrino-mass matrix of the form

$$m_{\text{eff}} = m_\nu^D \cdot (M_{\nu_R})^{-1} \cdot m_\nu^{D^T}. \quad (3)$$

Neutrino-flavour mixing [23] may then occur through either the Dirac matrix m_ν^D and/or the Majorana mass matrix M_{ν_R} , which may also feed flavour violation through to the charged leptons. In non-supersymmetric models with

massive neutrinos, the amplitudes for charged-lepton-flavour violation are proportional to inverse powers of the right-handed neutrino-mass scale M_{ν_R} , and thus the rates for rare decays are extremely suppressed [2]. On the other hand, in supersymmetric models one must also take the dynamics of sneutrinos $\tilde{\nu}$ into account, and these processes may only be only suppressed by inverse powers of the supersymmetry-breaking scale, which characterizes $m_{\tilde{\nu}}$ and is at most $\mathcal{O}(1)$ TeV [3].

The magnitudes of the predicted rates depend on the details of the masses and mixings of sparticles including the sneutrinos. If their soft supersymmetry-breaking masses are non-universal at M_{GUT} , large rates are in general predicted [24]. However, even if the soft supersymmetry-breaking masses are universal at M_{GUT} , renormalization effects in the MSSM with right-handed neutrinos spoil this diagonal form [3–5] at lower scales. This is because the Dirac neutrino and charged-lepton Yukawa couplings and masses m_ℓ cannot, in general, be diagonalised simultaneously. Since both these sets of lepton Yukawa couplings appear in the renormalization-group equations, the lepton and slepton mass matrices also may not be diagonalised simultaneously at low energies.

To illustrate this point, let us consider the simplest example of a model based on Abelian flavour symmetries and symmetric mass matrices [25]. Requiring large (2-3) mixing in this model [26] severely constrains the possible flavour charges and thus the forms of the charged-lepton and the neutrino-mass matrices. A representative example is given by Ansatz A of [5]:

$$m_\ell \propto \begin{pmatrix} \bar{\epsilon}^7 & \bar{\epsilon}^3 & \bar{\epsilon}^{7/2} \\ \bar{\epsilon}^3 & \bar{\epsilon} & \bar{\epsilon}^{1/2} \\ \bar{\epsilon}^{7/2} & \bar{\epsilon}^{1/2} & 1 \end{pmatrix}, \quad m_\nu^D \propto \begin{pmatrix} \epsilon^7 & \epsilon^3 & \epsilon^{7/2} \\ \epsilon^3 & \epsilon & \epsilon^{1/2} \\ \epsilon^{7/2} & \epsilon^{1/2} & 1 \end{pmatrix}, \quad (4)$$

where $\bar{\epsilon} = \epsilon^{1/2} = 0.2$. We already see that the two matrices cannot be simultaneously diagonalised and indeed,

$$V_\ell = \begin{pmatrix} 1 & \bar{\epsilon}^2 & -\bar{\epsilon}^{7/2} \\ -\bar{\epsilon}^2 & 1 & \bar{\epsilon}^{1/2} \\ \bar{\epsilon}^{7/2} & -\bar{\epsilon}^{1/2} & 1 \end{pmatrix}, \quad V_{\nu_D} = \begin{pmatrix} 1 & \bar{\epsilon}^4 & -\bar{\epsilon}^7 \\ -\bar{\epsilon}^4 & 1 & \bar{\epsilon} \\ \bar{\epsilon}^7 & -\bar{\epsilon} & 1 \end{pmatrix}. \quad (5)$$

Within this general framework, there is ambiguity in the specification of numerical coefficients in the matrix elements, which are expected to be of order unity. We return later to this point.

For a generic texture where the charged-lepton and neutrino matrices are not simultaneously diagonal, the slepton mass matrix acquires non-diagonal contributions from renormalization at scales below M_{GUT} . In the basis

where m_ℓ is diagonal, these corrections take the form [3]

$$\delta m_\ell^2 \propto \frac{1}{16\pi^2} (3 + a^2) \ln \frac{M_{\text{GUT}}}{M_N} \lambda_D^\dagger \lambda_D m_{3/2}^2, \quad (6)$$

where λ_D is the Dirac neutrino Yukawa coupling, M_N is the scale where the effective neutrino-mass operator is formed, a is related to the trilinear mass parameter $A_\ell \equiv a m_{3/2}$, and $m_{3/2}$ is the presumed common value m_0 of the scalar masses at the GUT scale.

In the case of non-universal soft masses, these corrections are generically negligible. However, the rates for $\Delta L \neq 0$ processes are generally too large in such non-universal models [5]. On the other hand, models with scalar-mass universality at M_{GUT} , such as no-scale [27] and gauge-mediated models [28], may yield acceptable rates for $\Delta L \neq 0$ processes. In such models, the contributions (6) related to neutrino masses dominate and lead to non-negligible rates for the lepton-flavour-violating processes which are determined by the off-diagonal terms in the Yukawa matrix λ_N . The various different solutions of the solar neutrino deficit [29,30], with a small/large mixing angle and with eV or ≈ 0.03 eV neutrinos, predict in general different rates for charged-lepton-flavour violation: the larger the μ - e mixing, and the larger the neutrino-mass scales that are required, the larger the rates. Thus, degenerate neutrinos with bimaximal mixing may be expected to yield significantly larger effects than, for instance, hierarchical neutrinos with a small vacuum mixing angle. Note, in particular, that the just-so solutions to the solar neutrino problem with $\delta m^2 \approx 10^{-10}$ eV² predict, in the case of hierarchical neutrino masses, rates that are small, even if the first/second-generation neutrino mixing is large.

In this MSSM framework, rare kaon decays are generated by box diagrams involving chargino and neutralino exchanges. For instance, for $K^0 \rightarrow \mu^\pm e^\mp$ we have the diagrams¹ of Fig. 1². It is clear that $K^{\pm,0} \rightarrow \pi^{\pm,0} \mu^\pm e^\mp$ decays can be generated in a similar way, but in this case the experimental bounds are worse by almost two orders of magnitude, and we do not discuss them in detail.

Our procedure for evaluating these contributions is as follows.

- (1) We first find the maximal squark mixing that is allowed by the neutral-kaon mass difference Δm_K [33].
- (2) We next find the maximal slepton mixing allowed by $\mu \rightarrow e\gamma$ and μ - e conversion in nuclei *in a model-independent way*.

¹ The contribution from the first diagram when the neutralino is a purely photino state has been discussed in [31]

² Kaon decays in left-right symmetric models have been analysed in [32]. In principle, using the mixing in the quark sector – in particular between s and d quarks – we can also generate $K \rightarrow \mu e$ by penguin diagrams. In the cases where all the quark mixing is in the down sector, or the right-handed mixing in the down sector is much bigger than the one in the left (which is the one bounded by V_{CKM}), the rates might be of relevance. However, we do not address here this model-dependent issue

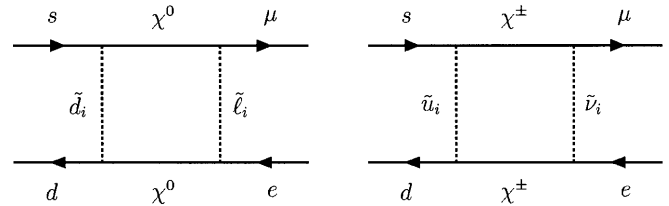


Fig. 1. MSSM box diagrams for $K^0 \rightarrow \mu^\pm e^\mp$. There is another neutralino exchange diagram corresponding to the permutation of the μ and e . Since χ^0 is a Majorana spinor, there are contributions from the neutralinos that differ in the number of mass insertions

- (3) Having fixed these values, finally we calculate the rates for rare kaon decays.

As we noted previously, the μ - e mixing is constrained by the form of the neutrino textures and thus by the recent neutrino data: in general, degenerate neutrinos with large angle MSW oscillations require smaller soft masses to be consistent with the observed rates. However, as we found in [5], even for the small-angle MSW solutions of the solar neutrino deficit, we can obtain large rates for values of m_0 and $m_{1/2}$ well below 500 GeV. The latest Super-Kamiokande data [34] on solar neutrinos favour large mixing angles, which might suggest larger μ - e flavour violation. Thus, considering models with small mixing angles is conservative.

For illustration, we focus here on one such model, namely variant A_1 of the texture (4) and (5), which has the numerical values of the ambiguous $\mathcal{O}(1)$ coefficients fixed as described in [5]. For definiteness, we choose its “inverted” option with negative off-diagonal entries in the Dirac neutrino coupling matrix [5]. Our results are presented in Fig. 2, where we summarise our predictions for $\text{BR}(K \rightarrow \mu e)$, in association with the predictions for $\text{BR}(\mu \rightarrow e\gamma)$ and $\text{BR}(\mu - e)$ for different values of $\tan\beta$ and $m_{1/2}$. We parametrise the supersymmetric masses in terms of the universal GUT-scale parameters m_0 and $m_{1/2}$, for sfermions and gauginos respectively, and use the renormalization-group equations of the MSSM to calculate the low-energy sparticle masses. Other relevant free parameters of the MSSM are the trilinear coupling A (for which we start with the initial condition $A_0 = -m_{1/2}$), the sign of the Higgs mixing parameter μ , and the value of $\tan\beta$. Models with different signs of μ give similar results: here we assume $\mu < 0$.

As expected, the larger the value of $\tan\beta$ and the smaller the soft supersymmetric terms, the larger the branching ratios, apart from certain cancellations. In the case $\tan\beta = 10$ and $m_{1/2} = 250$ GeV, we see that, for the range $m_0 \geq 170$ GeV where $\text{BR}(\mu \rightarrow e\gamma)$ and $\text{BR}(\mu - e)$ conversions are consistent with the current experimental bounds (1), $\text{BR}(K \rightarrow \mu e)$ is at most $2 \cdot 10^{-18}$. However, for the same value of $m_{1/2}$, when $\tan\beta = 20$ we find a significantly larger branching ratio at the small values of $m_0 \sim 170$ GeV. Moreover, for smaller $m_{1/2} = 150$ GeV, we gain almost two orders of magnitude when we consider m_0 in the low-mass window between 100 and 150 GeV. We

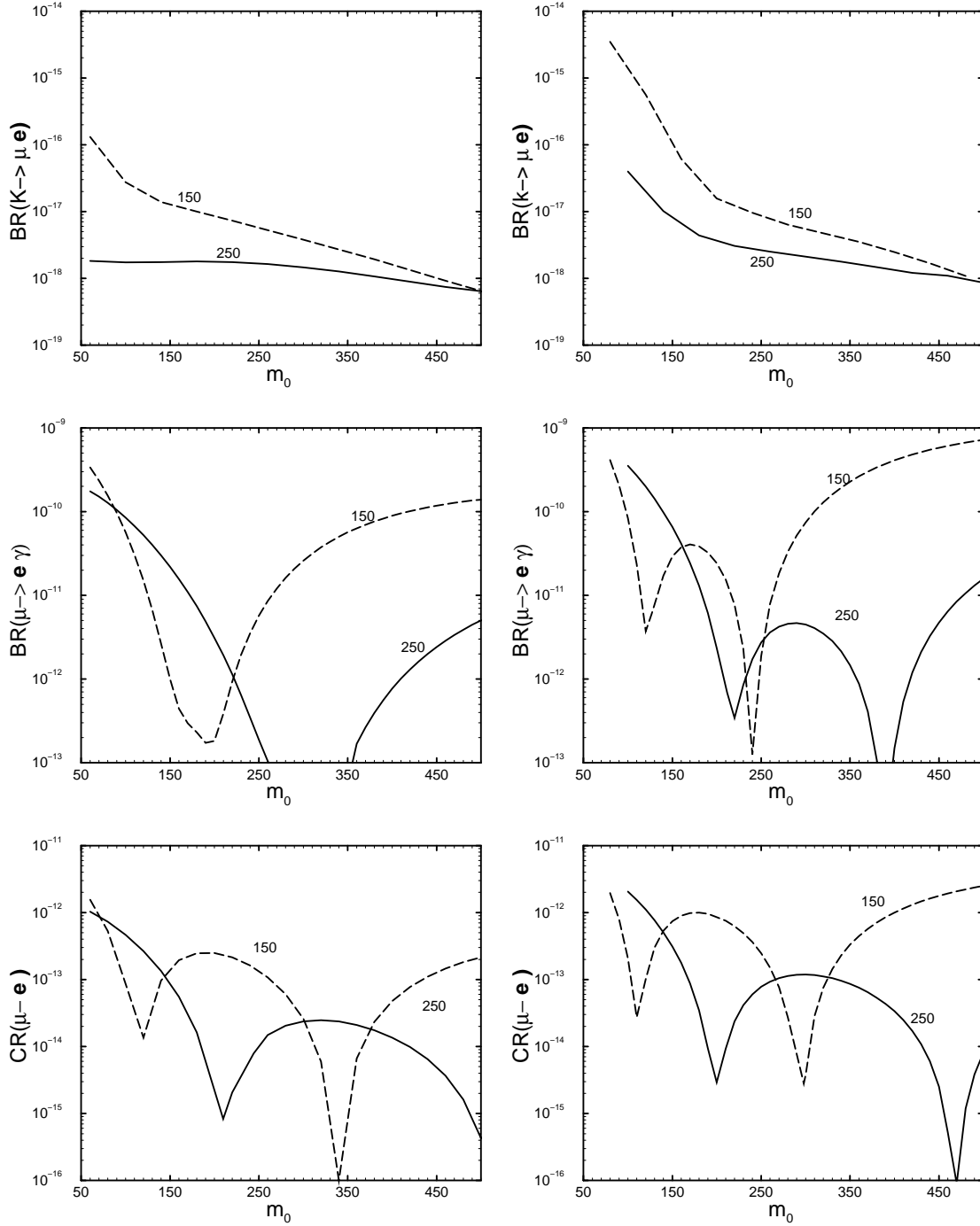


Fig. 2. Illustrative predictions for $BR(K \rightarrow \mu e)$, $BR(\mu \rightarrow e \gamma)$ and $BR(\mu - e)$ for different values of $\tan \beta = 10$ (left column), 20 (right column) and $m_{1/2} = 150$ (dashed lines), 250 GeV (solid lines), as functions of m_0 (in GeV)

recall [5] that these lower values of $m_{1/2}, m_0$ are consistent with accelerator constraints and generically yield cold dark matter densities in the range preferred by cosmology [35].

We do not discuss here other model textures for the mass matrices. Rather, our point here has been to demonstrate that, despite the limits from $\mu \rightarrow e \gamma$, $\mu - e$ conversion and Δm_K , the branching ratio of $K \rightarrow \mu e$ may be within

the reach of the next generation of experiments, namely in the range $10^{-16} \rightarrow 10^{-18}$, at least if $\tan \beta$ is large and the soft supersymmetry-breaking terms are small. The sensitivities (1) to $\mu \rightarrow e \gamma$ and $\mu - e$ conversion could be improved significantly even before a neutrino factory comes into operation, and such a machine would offer enhanced prospects for probing them. It is therefore likely that the best prospects for discovering charged-lepton-flavour vi-

olation may be offered by μ - e conversion and $\mu \rightarrow e\gamma$. However, rare kaon decays provide a complementary probe which also gives information on the squark mixing, in the context of the MSSM.

3 Rare kaon decays in R -violating supersymmetry

We now discuss kaon decays violating charged-lepton flavour beyond the context of the MSSM. As is well known, the gauge symmetries of the MSSM allow additional dimension-four Yukawa couplings, of the form

$$\lambda L_i L_j \bar{E}_k, \quad \lambda' L_i Q_j \bar{D}_k, \quad \lambda'' \bar{U}_i \bar{D}_j \bar{D}_k,$$

where the $L(Q)$ are the left-handed lepton (quark) superfields, and the $\bar{E}(\bar{D}, \bar{U})$ are the corresponding right-handed fields. If all these couplings were present simultaneously in the low-energy Lagrangian, they would generate unacceptably fast proton decay. Therefore, extra symmetries must be invoked to forbid all (R parity [36]), or subsets (baryon and lepton parities [37]) of these couplings. In the latter case, very interesting baryon- and lepton-number-violating processes may occur [38].

Imposing electroweak $SU(2)$ and colour $SU(3)$ invariance, one finds that there are just 45 R -violating couplings in total. Besides proton decay, there are many experimental constraints on these couplings, both individually and in various combinations, from the non-observation of modifications to standard model processes and of possible exotic processes [39, 40]. In order to understand the possible hierarchies of R -violating couplings, models of flavour symmetries have been invoked [42]. For instance, it was found in a previous work [43] that theories with symmetric fermion mass textures lead to the expectation that R -violating couplings are small: $\leq 10^{-3}$ - 10^{-4} for 100 GeV sfermion masses, whilst, in models with asymmetric fermion mass textures, dominance by a single coupling may be permitted, without however excluding several products of couplings from being non-negligible.

In this paper, therefore, we allow for the general possibility that several R -violating operators may be large, and discuss the limits on their combinations that are obtainable from kaon decays. In this class of models, whilst $\mu \rightarrow e\gamma$ occurs at the one-loop level, $\mu \rightarrow 3e$, μ - e conversion, $K^0 \rightarrow \mu^\pm e^\mp$ and $K^{\pm,0} \rightarrow \pi^{\pm,0} \mu^\pm e^\mp$ may occur at tree level via different combinations of couplings. For instance, in the case of $LQ\bar{D}$ couplings, $\mu \rightarrow 3e$ gives the limit [39]

$$(L_1 Q_i \bar{D}_j)(L_2 Q_i \bar{D}_j) \leq 10^{-4} \left(\frac{m_{\tilde{f}}}{100 \text{ GeV}} \right)^2, \quad (7)$$

whilst μ - e conversion in titanium gives

$$(L_2 Q_1 \bar{D}_k)(L_1 Q_1 \bar{D}_k) \leq 10^{-8} \left(\frac{m_{\tilde{f}}}{100 \text{ GeV}} \right)^2, \quad (8)$$

$$(L_2 Q_j \bar{D}_1)(L_1 Q_j \bar{D}_1) \leq 10^{-8} \left(\frac{m_{\tilde{f}}}{100 \text{ GeV}} \right)^2, \quad (9)$$

and the bound

$$(L_i Q_1 \bar{D}_2)(L_i Q_2 \bar{D}_1) \leq 10^{-9} \left(\frac{m_{\tilde{f}}}{100 \text{ GeV}} \right)^2 \quad (10)$$

is obtainable from Δm_K [40].

What is the connection of these results with neutrino masses? If R parity is violated, neutrino masses are generated via one-loop diagrams involving the vertices $\nu_i \bar{d}_k \tilde{d}_j$, $\bar{\nu}_i^c d_j \tilde{d}_k^*$ for $LQ\bar{D}$ operators and the vertices $\nu_i \bar{e}_k \tilde{\ell}_j$, $\bar{\nu}_i^c \ell_j \tilde{e}_k^*$ for $LL\bar{E}$ operators. Focusing on $LQ\bar{D}$ couplings and assuming that the left-right squark soft mixing terms are diagonal in the physical basis and proportional to the associated quark mass, the induced masses are given by

$$m_{\nu_{ii'}} \simeq \frac{n_c \lambda'_{ijk} \lambda'_{i'kj}}{16\pi^2} m_{d_j} m_{d_k} \times \left(\frac{f(m_{d_j}^2/m_{\tilde{d}_k}^2)}{m_{\tilde{d}_k}} + \frac{f(m_{d_k}^2/m_{\tilde{d}_j}^2)}{m_{\tilde{d}_j}} \right), \quad (11)$$

where $f(x) = (x \ln x - x + 1)/(x - 1)^2$, m_{d_i} is the down quark mass of the i th generation inside the loop, $m_{\tilde{d}_i}$ is an average of \tilde{d}_{Li} and \tilde{d}_{Ri} squark masses, and $n_c = 3$ is the colour factor.

Requiring that these contributions be consistent with the neutrino data gives bounds on the associated R -violating products. We can see from the above expression that the heavier the fermions in the loop (including the associated fermion mass arising from the soft mixing term), the stricter the bounds [44]. For example, demanding $m_{e\mu} < 2.5 \text{ eV}$ for sparticle masses of 100 GeV leads to $\lambda'_{133} \lambda'_{233} \leq 3.8 \cdot 10^{-7}$, whilst for $\lambda'_{122} \lambda'_{222}$ the bound drops to $2.3 \cdot 10^{-4}$ [44]. For higher sfermion masses, larger R -violating couplings are allowed. In Super-Kamiokande-friendly solutions with hierarchical neutrinos the bounds are stricter by two orders of magnitude.

Note, however, that neutrino masses do not strictly constrain $K \rightarrow \mu e$ (and in certain cases the rest of the flavour-violating processes), since

- (1) neutrino masses may only constrain products of $LL\bar{E}$ or $LQ\bar{D}$ operators, not mixed $LL\bar{E}$ - $LQ\bar{D}$ products;
- (2) even for the diagrams with products of only $LQ\bar{D}$ operators, rare kaon decays involve quarks of the lightest and second-lightest generations. In this case the bounds from neutrino masses are significantly weaker, and the stricter limits come from the current measurements of the rare kaon decays themselves. The same is true for $\mu \rightarrow e$ conversion and even for $\mu \rightarrow e\gamma$ via $LL\bar{E}$ couplings. For fermions of the first two generations, the bounds from the lepton-flavour-violating processes themselves tend to dominate.

Two-body K^0 decays to muons and electrons proceed via the diagrams shown in Fig. 3.

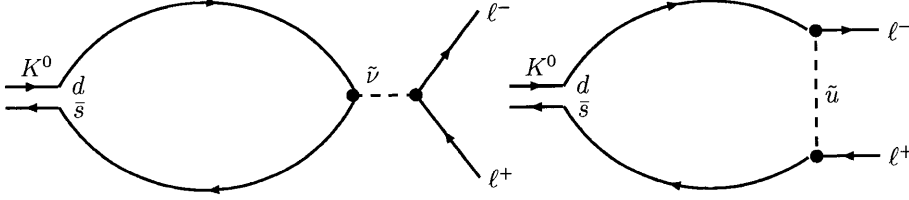


Fig. 3. Quark/sfermion diagrams involving R -violating couplings that yield two-body $K^0 \rightarrow \ell^\pm \ell^\mp$ decays

At the quark level, the effective Lagrangian for such processes has the form³ [40]

$$\begin{aligned} \mathcal{L}_{d\bar{s} \rightarrow \ell_j^- \ell_k^+} &= \frac{1}{m_{\tilde{\nu}_i}^2} \left[\lambda_{ijk}^* \lambda'_{i12} (\overline{s_R} d_L) (\overline{\ell_{jL}} \ell_{kR}) \right. \\ &\quad \left. + \lambda_{ikj} \lambda'_{i21} (\overline{s_L} d_R) (\overline{\ell_{jR}} \ell_{kL}) \right], \\ &\quad - \frac{\lambda_{ji1}^* \lambda'_{ki2}}{2m_{\tilde{u}_i}^2} (\overline{s_R} \gamma_\mu d_R) (\overline{\ell_{jL}} \gamma^\mu \ell_{kL}). \end{aligned} \quad (12)$$

The two different contributions from s -channel and t -channel diagrams put limits on different couplings [40], as we discuss below.

We have derived Feynman rules for the relevant effective kaon, pion and lepton interactions. Based on these Feynman rules, we have recalculated the important kaon decay processes, and update the limits on the products of R -violating couplings using the present experimental limits (2). The diagrams of Fig. 3 lead to the the following effective Lagrangian for $K^0 \ell^+ \ell^-$ interactions:

$$\begin{aligned} \mathcal{L}_{K^0 \ell_j^- \ell_k^+} &= \frac{F_{K^0}}{2m_{\tilde{\nu}_i}^2} \\ &\quad \times [\lambda_{ijk}^* \lambda'_{i12} (\overline{\ell_{jL}} \ell_{kR}) - \lambda_{ikj} \lambda'_{i21} (\overline{\ell_{jR}} \ell_{kL})] K^0(p_K) \\ &\quad - \frac{f_K}{4m_{\tilde{u}_i}^2} \lambda_{ji1}^* \lambda'_{ki2} p_K^\mu (\overline{\ell_{jL}} \gamma_\mu \ell_{kL}) K^0(p_K), \end{aligned} \quad (13)$$

where $F_{K^0} = m_{K^0}^2 f_K / (m_s + m_d)$, $m_s + m_d \simeq 0.15$ GeV is the sum of the current masses of the s and d quarks, and $f_K = 0.1598$ GeV is the kaon decay constant. The value of F_{K^0} is related to the pseudoscalar $\langle 0 | \bar{s} \gamma^5 d | K^0 \rangle = -F_{K^0}$ matrix element, and is obtained from f_K by using the Dirac equations for quarks. All QCD corrections are included in this phenomenological approach. In the following, we assume that the R -violating couplings are real and that only one of their products in (12) is non-zero.

We have implemented the Feynman rules in the CompHEP package [45], using the effective Lagrangian (13), and have obtained the following results:

$$\begin{aligned} \Gamma_{K^0 \rightarrow \ell_j^- \ell_k^+}(\tilde{\nu}_i) &= \frac{(\lambda_{ijk} \lambda'_{i12})^2 F_{K^0}^2}{64\pi m_{\tilde{\nu}_i}^4 m_{K^0}} \\ &\quad \times \left(1 - \frac{m_{\ell_j}^2 + m_{\ell_k}^2}{m_{K^0}^2} \right) \Delta(m_{K^0}, m_{\ell_j}, m_{\ell_k}), \end{aligned} \quad (14)$$

$$\Gamma_{K^0 \rightarrow \ell_j^- \ell_k^+}(\tilde{u}_i) = \frac{(\lambda'_{ji1} \lambda'_{ki2})^2 f_K^2}{256\pi m_{\tilde{u}_i}^4 m_{K^0}} \quad (15)$$

³ Note that, for energies of the order of the kaon mass, both s - and t -channel diagrams yield contact interactions

$$\times \left(m_{\ell_j}^2 + m_{\ell_k}^2 - \frac{(m_{\ell_j}^2 - m_{\ell_k}^2)^2}{m_{K^0}^2} \right) \Delta(m_{K^0}, m_{\ell_j}, m_{\ell_k}),$$

where $\Delta(a, b, c) = ([a^2 - (b+c)^2][a^2 - (b-c)^2])^{1/2}$ is the triangle function. As there exist two similar contributions in the s -channel, coming from terms with different couplings $\lambda_{ijk} \lambda'_{i12}$ and $\lambda_{ikj} \lambda'_{i21}$, as seen in the first line in (13), we give only one of them in (14).

We obtain the following nominal numerical results for $K^0 \rightarrow \ell^+ \ell^-$ decay via $\tilde{\nu}$ exchange, and the corresponding limits on the $\lambda \lambda'$ products (for the numerical results for $\Gamma_{K^0 \rightarrow \ell^+ \ell^-}$ we have used the nominal values $\lambda = \lambda' = 1$ and $m_{\tilde{\nu}(\tilde{u})} = 100$ GeV):

$$\begin{aligned} \Gamma_{K^0 \rightarrow e^+ \mu^-} &= 1.57 \times 10^{-12} \text{ GeV}, \\ \lambda_{i21} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 6.2 \times 10^{-9}, \\ \lambda_{i12} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 6.2 \times 10^{-9}, \\ \Gamma_{K^0 \rightarrow e^- \mu^+} &= 1.57 \times 10^{-12} \text{ GeV}, \\ \lambda_{i12} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 6.2 \times 10^{-9}, \\ \lambda_{i21} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 6.2 \times 10^{-9}, \\ \Gamma_{K^0 \rightarrow e^+ e^-} &= 1.72 \times 10^{-12} \text{ GeV}, \\ \lambda_{i11} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 1.0 \times 10^{-8}, \\ \lambda_{i11} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 1.0 \times 10^{-8}, \\ \Gamma_{K^0 \rightarrow \mu^+ \mu^-} &= 1.42 \times 10^{-12} \text{ GeV}, \\ \lambda_{i22} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 2.6 \times 10^{-7}, \\ \lambda_{i22} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 2.6 \times 10^{-7}. \end{aligned} \quad (16)$$

For $K^0 \rightarrow \ell^+ \ell^-$ decay via up-squark exchange we have the following limits:

$$\begin{aligned} \Gamma_{K^0 \rightarrow e^+ \mu^-} &= 1.61 \times 10^{-15} \text{ GeV}, \\ \lambda'_{2i1} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}} \right)^2 &\leq 1.9 \times 10^{-7}, \\ \Gamma_{K^0 \rightarrow e^- \mu^+} &= 1.61 \times 10^{-15} \text{ GeV}, \\ \lambda'_{1i1} \lambda'_{i22} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}} \right)^2 &\leq 1.9 \times 10^{-7}, \end{aligned}$$

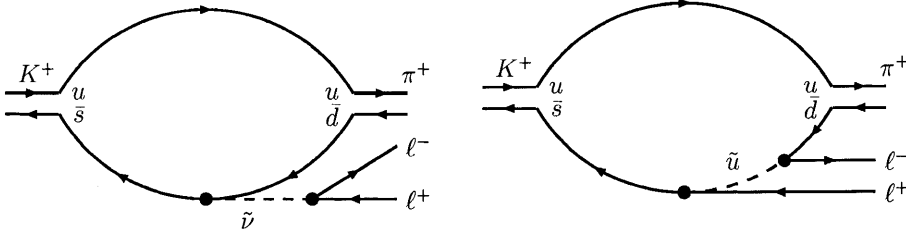


Fig. 4. Diagrams involving R -violating couplings that yield the three-body leptonic decays $K^+ \rightarrow \pi^+ \ell^- \ell^+$

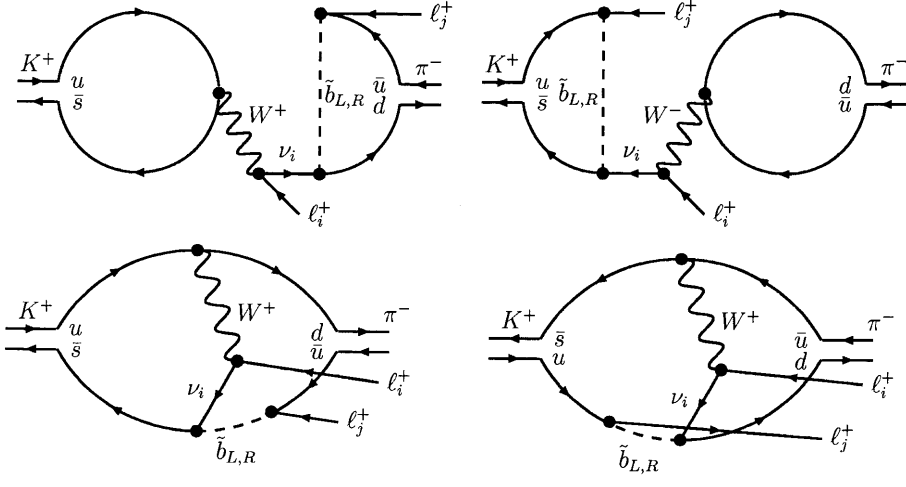


Fig. 5. Diagrams involving R -violating couplings that yield three-body like-sign leptons decays $K^+ \rightarrow \pi^- \ell_i^+ \ell_j^+$

$$\begin{aligned}
\Gamma_{K^0 \rightarrow e^+ e^-} &= 8.25 \times 10^{-20} \text{ GeV}, \\
\lambda'_{1i1} \lambda'_{1i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}} \right)^2 &\leq 4.7 \times 10^{-5}, \\
\Gamma_{K^0 \rightarrow \mu^+ \mu^-} &= 3.19 \times 10^{-15} \text{ GeV}, \\
\lambda'_{2i1} \lambda'_{2i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}} \right)^2 &\leq 5.4 \times 10^{-6}. \quad (17)
\end{aligned}$$

We have used in our calculations the decay width $\Gamma_{\text{exp}}(K_L^0) = 1.273 \times 10^{-17} \text{ GeV}$ and the experimental limits on $K^0 \rightarrow \ell^+ \ell^-$ decay widths shown in (2).

We now discuss the diagrams for 3-body kaon decays to pions and two charged leptons, of which there are two qualitatively different kinds.

- (1) The kaon may decay into a pion of the same charge, in which case the leptons in the final state must have opposite signs: $K^\pm \rightarrow \pi^\pm \ell^\mp \ell'^\pm$ and $K^0 \rightarrow \pi^0 \ell^\mp \ell'^\pm$. The corresponding diagram for the first process is shown in Fig. 4.
- (2) The kaon may decay into a pion with the opposite charge, in which case the leptons in the final state must have the same signs: $K^\pm \rightarrow \pi^\mp \ell^\pm \ell'^\pm$. Representative diagrams for this process are shown in Fig. 5. This process involves two heavy virtual particles, the W boson and a down squark. One should note that decay width of this process is directly proportional to the mixing between the left- and right-handed squark states, denoted by \tilde{b}_L and \tilde{b}_R , respectively. If there is no mixing, this same-sign-lepton process mentioned vanishes. One can expect sizeable mixing only for squarks of the third generation (and especially in the

high-tan β region), which is why we have used $\tilde{b}_{L,R}$ in the diagram.

The possibilities for $K^\pm \rightarrow \pi^\pm \ell^\mp \ell'^\pm$ or $K^0 \rightarrow \pi^0 \ell^\mp \ell'^\pm$ decay may be further subdivided into two groups.

- (1) Diagrams involving only squarks, via which experimental upper limits bound products of $LQ\bar{D}$ operators.
- (2) Diagrams involving also sleptons, which yield bounds on products of $LQ\bar{D}$ and $LL\bar{E}$ operators.

The diagrams of Fig. 4 lead to the the following effective Lagrangian for $K^+ \pi^- \ell^+ \ell^-$ interactions:

$$\begin{aligned}
\mathcal{L}_{K^+ \pi^- \ell_k^- \ell_j^+} &= \frac{M_s}{m_{\tilde{\nu}_i}^2} \\
&\times [\lambda_{ijk}^* \lambda'_{i12} (\overline{\ell_{jL}} \ell_{kR}) + \lambda_{ikj} \lambda_{i21}^* (\overline{\ell_{jR}} \ell_{kL})] \\
&\times K^+(p_K) \pi^-(p_\pi), \\
&+ \frac{f_+}{4m_{\tilde{u}_i}^2} \lambda_{ji1}^* \lambda'_{ki2} \\
&\times (p_K + p_\pi)^\mu (\overline{\ell_{jL}} \gamma_\mu \ell_{kL}) K^+(p_K) \pi^-(p_\pi), \quad (18)
\end{aligned}$$

where $M_s \simeq 0.49 \text{ GeV}$ is the constituent mass of the s quark. In the general case, the vector matrix element of the $K \rightarrow \pi$ transition is parametrised by two momentum-dependent form factors $f_+(t)$ and $f_-(t)$:

$$\begin{aligned}
\langle 0 | \bar{s} \gamma^\mu d | K^+(p_K) \pi^-(p_\pi) \rangle \\
&= \langle \pi^+(p_\pi) | \bar{s} \gamma^\mu d | K^+(p_K) \rangle \\
&= (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t),
\end{aligned}$$

where $t = (p_K - p_\pi)^2$ is the squared momentum transferred to the lepton pair. The experimental data on semileptonic

kaon decay are adequately described by a linear approximation for f_+ :

$$f_+(t) = f_+(0) [1 + \lambda_+(t/m_\pi^2)],$$

where $\lambda_+ \simeq 0.03$ and $f_- \simeq -0.31 \pm 0.15$. In the case of exact flavour $SU(3)$ symmetry, the following relations hold: $f_+(0) = 1$ and $f_-(0) = 0$. Due to the Ademollo–Gatto theorem, the first form factor is renormalized only at second order in the $SU(3)$ -violating interactions, and is therefore expected to be close to unity. The Ademollo–Gatto theorem is not applicable to f_- , whose value is close to zero, as follows from experiments and the Callan–Treiman relation:

$$f_-(m_K^2) = \frac{f_K}{f_\pi} - f_+(m_K^2) \simeq -0.15.$$

For our estimations, we set $f_+(0) = 1$ and neglect f_- . To estimate the scalar form factor, we use the relativistic quark model, which gives [41]

$$\langle 0 | \bar{s}d | K^+(p_K) \pi^-(-p_\pi) \rangle = \langle \pi^+(p_\pi) | \bar{s}d | K^+(p_K) \rangle \simeq -2M_s,$$

where we keep only the leading term and drop the momentum dependence, and $M_s \simeq 0.49 \text{ GeV}$ as before. In the approximation of unbroken $SU(3)$ symmetry, the corresponding form factors for the neutral kaons are smaller by a factor of $2^{1/2}$.

We can now estimate the decay rates of the charged kaons:

$$\begin{aligned} \Gamma_{K^+ \rightarrow \pi^+ \ell_j^- \ell_k^+}(\tilde{\nu}_i) &= \frac{(\lambda_{ijk} \lambda'_{i12})^2 M_s^2}{256 \pi^3 m_{\tilde{\nu}_i}^4 m_{K^+}^3} \\ &\times \int_{(m_{\ell_j} + m_{\ell_k})^2}^{(m_{K^+} - m_{\pi^+})^2} \Delta(t, m_{\ell_j}, m_{\ell_k}) \Delta(t, m_{K^+}, m_{\pi^+}) \\ &\times \left(t^2 - m_{\ell_j}^2 - m_{\ell_k}^2 \right) \frac{dt^2}{t^2}, \end{aligned} \quad (19)$$

$$\begin{aligned} \Gamma_{K^+ \rightarrow \pi^+ \ell_j^- \ell_k^+}(\tilde{u}_i) &= \frac{(\lambda'_{ji1} \lambda'_{ki2})^2}{2048 \pi^3 m_{\tilde{u}_i}^4 m_{K^+}^3} \\ &\times \int_{(m_{\ell_j} + m_{\ell_k})^2}^{(m_{K^+} - m_{\pi^+})^2} \Delta(t, m_{\ell_j}, m_{\ell_k}) \Delta(t, m_{K^+}, m_{\pi^+}) \\ &\times \left[\left(m_{K^+}^2 + m_{\pi^+}^2 + m_{\ell_j}^2 + m_{\ell_k}^2 - t^2 \right) F_1(t) - F_2(t) \right. \\ &\left. + \frac{m_{\ell_j}^2 + m_{\ell_k}^2}{2} \left(t^2 - m_{\ell_j}^2 - m_{\ell_k}^2 \right) - 2m_{\pi^+}^2 m_{K^+}^2 \right] \\ &\times [1 + \lambda_+(t^2/m_\pi^2)]^2 \frac{dt^2}{t^2}, \end{aligned} \quad (20)$$

where $F_1(t) = q_{\max}(t) + q_{\min}(t)$, $F_2(t) = 2(q_{\max}^2(t) + q_{\min}^2(t)q_{\min}(t) + q_{\min}^2(t))/3$, and

$$q_{\max}(t) = \frac{1}{4t^2} \left[\left(m_{K^+}^2 - m_{\pi^+}^2 + m_{\ell_j}^2 - m_{\ell_k}^2 \right)^2 \right.$$

$$\begin{aligned} &\left. - \left(\Delta(t, m_{\ell_j}, m_{\ell_k}) - \Delta(t, m_{K^+}, m_{\pi^+}) \right)^2 \right], \\ q_{\min}(t) &= \frac{1}{4t^2} \left[\left(m_{K^+}^2 - m_{\pi^+}^2 + m_{\ell_j}^2 - m_{\ell_k}^2 \right)^2 \right. \\ &\left. - \left(\Delta(t, m_{\ell_j}, m_{\ell_k}) + \Delta(t, m_{K^+}, m_{\pi^+}) \right)^2 \right]. \end{aligned}$$

For the decay rates of the neutral kaons, the $SU(3)$ factor of $1/2$ and the corresponding masses for the neutral mesons m_{K^0} and m_{π^0} must be taken into account.

We obtain the following nominal numerical results for $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ and $K^0 \rightarrow \pi^0 \ell^+ \ell^-$ decays via $\tilde{\nu}$ exchange, and the corresponding limits on the $\lambda\lambda'$ products (for the numerical results we have used the nominal values $\lambda = \lambda' = 1$ and $m_{\tilde{\nu}(\tilde{u})} = 100 \text{ GeV}$):

$$\begin{aligned} \Gamma_{K^+ \rightarrow \pi^+ e^+ \mu^-} &= 1.38 \times 10^{-15} \text{ GeV}, \\ \lambda_{i21} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 4.5 \times 10^{-6}, \\ \lambda_{i12} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 4.5 \times 10^{-6}, \\ \Gamma_{K^0 \rightarrow \pi^0 e^+ \mu^-} &= 7.71 \times 10^{-16} \text{ GeV}, \\ \lambda_{i21} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 7.2 \times 10^{-6}, \\ \lambda_{i12} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 7.2 \times 10^{-6}, \\ \Gamma_{K^+ \rightarrow \pi^+ e^- \mu^+} &= 1.38 \times 10^{-15} \text{ GeV}, \\ \lambda_{i12} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 1.0 \times 10^{-6}, \\ \lambda_{i21} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 1.0 \times 10^{-6}, \\ \Gamma_{K^0 \rightarrow \pi^0 e^- \mu^+} &= 7.71 \times 10^{-16} \text{ GeV}, \\ \lambda_{i12} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 7.2 \times 10^{-6}, \\ \lambda_{i21} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 7.2 \times 10^{-6}, \\ \Gamma_{K^+ \rightarrow \pi^+ e^+ e^-} &= 2.14 \times 10^{-15} \text{ GeV}, \\ \lambda_{i11} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 8.8 \times 10^{-5}, \\ \lambda_{i11} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 8.8 \times 10^{-5}, \\ \Gamma_{K^0 \rightarrow \pi^0 e^+ e^-} &= 1.17 \times 10^{-15} \text{ GeV}, \\ \lambda_{i11} \lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 6.8 \times 10^{-6}, \\ \lambda_{i11} \lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}} \right)^2 &\leq 6.8 \times 10^{-6}, \\ \Gamma_{K^+ \rightarrow \pi^+ \mu^+ \mu^-} &= 7.58 \times 10^{-16} \text{ GeV}, \end{aligned}$$

$$\begin{aligned}
\lambda_{i22}\lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}}\right)^2 &\leq 8.2 \times 10^{-5}, \\
\lambda_{i22}\lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}}\right)^2 &\leq 8.2 \times 10^{-5}, \\
\Gamma_{K^0 \rightarrow \pi^0 \mu^+ \mu^-} &= 4.38 \times 10^{-16} \text{ GeV}, \\
\lambda_{i22}\lambda'_{i12} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}}\right)^2 &\leq 1.2 \times 10^{-5}, \\
\lambda_{i22}\lambda'_{i21} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}}\right)^2 &\leq 1.2 \times 10^{-5}. \quad (21)
\end{aligned}$$

For the decays via up-squark exchange we have the following limits:

$$\begin{aligned}
\Gamma_{K^+ \rightarrow \pi^+ e^+ \mu^-} &= 1.61 \times 10^{-16} \text{ GeV}, \\
\lambda'_{2i1}\lambda'_{1i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 1.3 \times 10^{-5}, \\
\Gamma_{K^0 \rightarrow \pi^0 e^+ \mu^-} &= 9.06 \times 10^{-17} \text{ GeV}, \\
\lambda'_{2i1}\lambda'_{1i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 2.1 \times 10^{-5}, \\
\Gamma_{K^+ \rightarrow \pi^+ e^- \mu^+} &= 1.61 \times 10^{-16} \text{ GeV}, \\
\lambda'_{1i1}\lambda'_{2i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 3.0 \times 10^{-6}, \\
\Gamma_{K^0 \rightarrow \pi^0 e^- \mu^+} &= 9.06 \times 10^{-17} \text{ GeV}, \\
\lambda'_{1i1}\lambda'_{2i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 2.1 \times 10^{-5}, \\
\Gamma_{K^+ \rightarrow \pi^+ e^+ e^-} &= 2.38 \times 10^{-16} \text{ GeV}, \\
\lambda'_{1i1}\lambda'_{1i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 2.7 \times 10^{-4}, \\
\Gamma_{K^0 \rightarrow \pi^0 e^+ e^-} &= 1.31 \times 10^{-16} \text{ GeV}, \\
\lambda'_{1i1}\lambda'_{1i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 2.0 \times 10^{-5}, \\
\Gamma_{K^+ \rightarrow \pi^+ \mu^+ \mu^-} &= 9.33 \times 10^{-17} \text{ GeV}, \\
\lambda'_{2i1}\lambda'_{2i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 2.3 \times 10^{-4}, \\
\Gamma_{K^0 \rightarrow \pi^0 \mu^+ \mu^-} &= 5.48 \times 10^{-17} \text{ GeV}, \\
\lambda'_{2i1}\lambda'_{2i2} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{u}}}\right)^2 &\leq 3.4 \times 10^{-5}. \quad (22)
\end{aligned}$$

We have used in our calculations the decay width $\Gamma_{\text{exp}}(K^+) = 5.314 \times 10^{-17} \text{ GeV}$ and the present limits on the $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ and $K_L^0 \rightarrow \pi^0 \ell^+ \ell^-$ decay widths (2). Note that there are different constraints from the $K^+ \rightarrow \pi^+ e^+ \mu^-$ and $K^+ \rightarrow \pi^+ e^- \mu^+$ decays, because of the rather different experimental limits $\text{BR}(K^+ \rightarrow \pi^+ e^+ \mu^-) \leq 6.9 \times 10^{-9}$ and $\text{BR}(K^+ \rightarrow \pi^+ e^- \mu^+) \leq 2.8 \times 10^{-11}$. The limit obtained from $K^0 \rightarrow \ell^+ \ell^-$ is typically 1–2 orders of magnitude better than that derived from $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay.

As already mentioned, diagrams with non-zero sbotom-quark mixing may lead to like-sign leptons, as seen

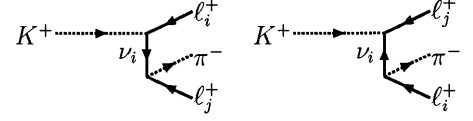


Fig. 6. Diagrams for the like-sign lepton decay $K^+ \rightarrow \pi^- \ell_i^+ \ell_j^+$, in terms of the effective standard-model-like interactions $K\ell\nu$ and $\pi\ell\nu$ and effective $K\ell^C\nu$ and $\pi\ell^C\nu$ interactions related to R -violating operators

in Fig. 5. They arise from the effective lepton-number-violating contact interactions

$$\mathcal{L} = -\frac{\lambda'_{ikp}\lambda'_{jqk}V_{\text{LR}}}{m_{\tilde{d}_k}^2} (\overline{d_{pR}}\nu_{iL}) \left((\overline{\ell_{jL}})^C u_{qL} \right) + \text{h.c.}, \quad (23)$$

where V_{LR} denotes left–right squark mixing matrix element.

There are two different topologies, shown in the first and second row of Fig. 5, respectively. Diagrams in the first row lead to an effective tree-level process, since the W boson and squark masses are much bigger than the typical energy scale, which is of the order of the kaon mass. Diagrams in the second row cannot be reduced to tree-level diagrams, since one has a neutrino propagator in the loop. However, this last set of diagrams give a contribution that is typically about 2 orders of magnitude lower than the diagrams of the first row [46]. Therefore, for the sake of simplicity, we neglect the second-row diagrams, and have derived an effective Lagrangian only for the first two diagrams of Fig. 5. Two kinds of effective interaction appear: standard-model-like $K\ell\nu$ and $\pi\ell\nu$ interactions and new effective interactions related to R -violating operators of the forms $K\ell^C\nu$ and $\pi\ell^C\nu$.

The effective Lagrangian for those interactions take the following forms:

$$\mathcal{L}_{K+\ell_i^- \bar{\nu}_i} = V_{us}\sqrt{2}G_{\text{F}}f_K p_K^\mu (\overline{\nu_{iL}}\gamma_\mu \ell_{iL}) K^+(p_K), \quad (24)$$

$$\mathcal{L}_{\pi+\ell_i^- \bar{\nu}_i} = V_{ud}\sqrt{2}G_{\text{F}}f_\pi p_\pi^\mu (\overline{\nu_{iL}}\gamma_\mu \ell_{iL}) \pi^+(p_\pi), \quad (25)$$

where f_K and $f_\pi = 0.1307 \text{ GeV}$ are the kaon and pion decay constants, respectively, $G_{\text{F}} = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant and V_{us}, V_{ud} are CKM matrix elements, and

$$\begin{aligned}
\mathcal{L}_{K+(\ell_j^-)^C \nu_i}(\tilde{d}_k) &= \frac{\lambda'_{ik2}\lambda'_{j1k}V_{\text{LR}}}{4m_{\tilde{d}_k}^2} \\
&\times F_{K^+} \left((\overline{\ell_{jL}})^C \nu_{iL} \right) K^+(p_K), \quad (26)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\pi+(\ell_j^-)^C \nu_i}(\tilde{d}_k) &= \frac{\lambda'_{ik1}\lambda'_{j1k}V_{\text{LR}}}{4m_{\tilde{d}_k}^2} \\
&\times F_{\pi^+} \left((\overline{\ell_{jL}})^C \nu_{iL} \right) \pi^+(p_\pi), \quad (27)
\end{aligned}$$

where $F_{K^+} = m_{K^+}^2 f_K / (m_s + m_u)$, $F_{\pi^+} = m_{\pi^+}^2 f_\pi / (m_d + m_u)$, $m_s + m_u \simeq 0.15 \text{ GeV}$, and $m_d + m_u \simeq 0.01 \text{ GeV}$.

The Feynman diagrams for $K^+ \rightarrow \pi^- \ell_i^+ \ell_j^+$ decay are given in terms of these effective interactions, as shown in Fig. 6. They yield the matrix elements

$$\mathcal{M} = \frac{\lambda'_{ik1} \lambda'_{j1k} V_{LR} F_{\pi^+}}{2m_{\tilde{d}_k}^2} \quad (28)$$

$$\times V_{us} \frac{G_F}{\sqrt{2}} f_K m_{\ell_i} \left((\ell_{jL})^C \frac{\hat{p}_{\nu_i}}{p_{\nu_i}^2} \ell_{iR} \right) K^+(p_K) \pi^-(p_\pi),$$

and

$$\mathcal{M} = \frac{\lambda'_{ik2} \lambda'_{j1k} V_{LR} F_{K^+}}{2m_{\tilde{d}_k}^2} \quad (29)$$

$$\times V_{ud} \frac{G_F}{\sqrt{2}} f_\pi m_{\ell_i} \left((\ell_{jL})^C \frac{\hat{p}_{\nu_i}}{p_{\nu_i}^2} \ell_{iR} \right) K^+(p_K) \pi^-(p_\pi).$$

Recalling that $V_{us}/V_{ud} \approx 0.2$, $F_{K^+} \approx F_{\pi^+}$ and $f_K \approx f_\pi$, the main contribution comes from the last matrix element with $i = 2$, because the matrix element is chirally suppressed in the other case. As an example, one can obtain a constraint on the product of $\lambda'_{2k2} \lambda'_{11k}$ and V_{LR} from $K^+ \rightarrow \pi^- e^+ \mu^+$ decay. In this case, we have the following numerical result⁴:

$$\Gamma(K^+ \rightarrow \pi^- e^+ \mu^+) = V_{LR}^2 (\lambda'_{2k2} \lambda'_{11k})^2$$

$$\times \left(\frac{100 \text{ GeV}}{m_{\tilde{d}_k}} \right)^4 2.5 \times 10^{-28} \text{ GeV}, \quad (30)$$

or

$$V_{LR} (\lambda'_{2k2} \lambda'_{11k}) \times \left(\frac{100 \text{ GeV}}{m_{\tilde{d}_k}} \right)^2 \leq 10. \quad (31)$$

It is apparent that kaon decay into a pion and a like-sign lepton pair is too strongly suppressed to be useful at present: the corresponding bounds on the $\lambda' \lambda'$ product are currently of the order of 100.

4 Conclusions

We have discussed in this paper flavour-violating decays of kaons into charged-lepton pairs in supersymmetric theories, in both the minimal supersymmetric standard model and R -violating models. In the first case, these decays are generated by box diagrams involving charginos and neutralinos, and both the squark and the slepton mixings enter in the analysis. The process looks promising for correlating the (s)quark and (s)lepton mixing by a combined study of rare charged lepton and kaon decays. Despite the limits from $\mu \rightarrow e\gamma$, μ - e conversion and Δm_K , the kaon decay branching ratios for large $\tan\beta$ and small soft supersymmetry-breaking terms may be accessible to a future generation of experiments using new intense proton sources.

⁴ In [47] a similar constraint was found with a different choice of diagrams

In the case of R -violating supersymmetry, such rare kaon decays may occur at tree level. In this case, $\mu \rightarrow e\gamma$ again occurs via one-loop diagrams, whilst μ - e conversion may also occur at tree-level, but via a set of operators different from those relevant to kaon decays. In this framework, we studied the expected rates for the decays $K \rightarrow \mu^\pm e^\mp$ and $K \rightarrow \pi \mu e$, for all two- and three-body processes. Using the current experimental data, we obtained the bounds on products of $LL\bar{E}$ and $LQ\bar{D}$ operators summarized in (16) and (17). We have also noted the possibility of like-sign lepton events in the presence of non-zero \tilde{b}_L - \tilde{b}_R mixing, but for this to occur at a significant rate one would need large R -violating couplings.

Our final conclusion is that lepton-flavour-violating rare kaon decays have the potential to provide important information on the issue of flavour physics. Any future observation would, in addition, help distinguish between different supersymmetric theories.

Acknowledgements. We thank Gerhard Buchalla for useful discussions.

Appendix

A The rate for $K \rightarrow \mu e$ decay via box diagrams in the MSSM

The branching ratio for $K \rightarrow \mu e$ is given by

$$\text{BR}(K \rightarrow \mu e) = \frac{2.65\lambda^2}{2G_F \sin^2 \theta_W} (|K_L|^2 + |K_R|^2), \quad (32)$$

where $\lambda = (m_d + m_s)m_\mu/m_K^2$, and K_L , K_R are given by the following expressions:

$$K_L = K_L^c + (K_L^{n(1)} + K_L^{n(2)}), \quad (33)$$

$$K_R = K_R^c + (K_R^{n(1)} + K_R^{n(2)}), \quad (34)$$

with

$$K_L^c = \frac{1}{4} J_{4(A,B,X,Y)} \left(-\frac{\lambda}{2} C_{dAX}^{R(d)} C_{sBX}^{R(d)*} C_{\mu BY}^{R(\ell)} C_{eAY}^{R(\ell)*} \right.$$

$$+ C_{dAX}^{R(d)} C_{sBX}^{L(d)*} C_{\mu BY}^{L(\ell)} C_{eAY}^{R(\ell)*} \left. \right)$$

$$- \frac{1}{4} I_{4(A,B,X,Y)} m_{\tilde{\chi}_A^-} m_{\tilde{\chi}_B^-} \left(C_{dAX}^{L(d)} C_{sBX}^{R(d)*} C_{\mu BY}^{L(\ell)} C_{eAY}^{R(\ell)*} \right.$$

$$+ \lambda C_{dAX}^{L(d)} C_{sBX}^{L(d)*} C_{\mu BY}^{R(\ell)} C_{eAY}^{R(\ell)*} \left. \right), \quad (35)$$

$$K_R^c = -K_L^c|_{L \leftrightarrow R}, \quad (36)$$

where $m_{\tilde{\chi}_{A,B}}$ and $m_{\tilde{l}_{X,Y}}$ denote chargino and sneutrino masses, in the chargino contribution. Moreover,

$$iJ_{4(A,B,X,Y)} = \int \frac{d^4 k}{(2\pi)^4} \quad (37)$$

$$\times \frac{k^2}{(k^2 - M_{\tilde{\chi}_A}^2)(k^2 - M_{\tilde{\chi}_B}^2)(k^2 - m_{\tilde{l}_X}^2)(k^2 - m_{\tilde{l}_Y}^2)},$$

$$iI_{4(A,B,X,Y)} = \int \frac{d^4k}{(2\pi)^4} \quad (38)$$

$$\times \frac{1}{(k^2 - M_{\tilde{\chi}_A}^2)(k^2 - M_{\tilde{\chi}_B}^2)(k^2 - m_{\tilde{l}_X}^2)(k^2 - m_{\tilde{l}_Y}^2)},$$

whilst the mixing coefficients $C^{R,L}$ appear in the fermion–sfermion–chargino interaction Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \bar{\ell}_i (C_{iAX}^{R(l)} P_R + C_{iAX}^{L(l)} P_L) \tilde{\chi}_A^- \tilde{\nu}_X \\ & + \bar{\nu}_i (C_{iAX}^{R(\nu)} P_R + C_{iAX}^{L(\nu)} P_L) \tilde{\chi}_A^+ \tilde{\ell}_X \\ & + \bar{d}_i (C_{iAX}^{R(d)} P_R + C_{iAX}^{L(d)} P_L) \tilde{\chi}_A^- \tilde{u}_X \\ & + \bar{u}_i (C_{iAX}^{R(u)} P_R + C_{iAX}^{L(u)} P_L) \tilde{\chi}_A^+ \tilde{d}_X + \text{h.c.}, \quad (39) \end{aligned}$$

and their explicit expressions are given, for instance, in [48].

The fermion–sfermion–neutralino interaction Lagrangian is similarly written as

$$\mathcal{L}_{\text{int}} = \bar{f}_i (N_{iAX}^{R(f)} P_R + N_{iAX}^{L(f)} P_L) \tilde{\chi}_A^0 \tilde{f}_X, \quad (40)$$

where f stands for l, ν, d and u . The neutralino box contributions corresponding to the permutations of the μ and e in the external lines, $K_L^n(2)$ and $K_R^n(2)$, are then found to be

$$\begin{aligned} K_L^{n(1)} = & \frac{1}{4} J_{4(A,B,X,Y)} \left(-\frac{\lambda}{2} N_{dAX}^{R(d)} N_{sBX}^{R(d)*} N_{\mu BY}^{R(\ell)} N_{eAY}^{R(\ell)*} \right. \\ & + N_{dAX}^{R(d)} N_{sBX}^{L(d)*} N_{\mu BY}^{L(\ell)} N_{eAY}^{R(\ell)*} \Big) \\ & - \frac{1}{4} I_{4(A,B,X,Y)} m_{\tilde{\chi}_A^0} m_{\tilde{\chi}_B^0} \left(N_{dAX}^{L(d)} N_{sBX}^{R(d)*} N_{\mu BY}^{L(\ell)} N_{eAY}^{R(\ell)*} \right. \\ & + \lambda N_{dAX}^{L(d)} N_{sBX}^{L(d)*} N_{\mu BY}^{R(\ell)} N_{eAY}^{R(\ell)*} \Big), \quad (41) \end{aligned}$$

$$K_R^{n(1)} = -K_L^{n(1)}|_{L \leftrightarrow R}, \quad (42)$$

and

$$\begin{aligned} K_L^{n(2)} = & \frac{1}{4} J_{4(A,B,X,Y)} \left(\frac{\lambda}{2} N_{dAX}^{R(d)} N_{sBX}^{R(d)*} N_{\mu BY}^{L(\ell)} N_{eAY}^{L(\ell)*} \right. \\ & - N_{dAX}^{R(d)} N_{sBX}^{L(d)*} N_{\mu BY}^{L(\ell)} N_{eAY}^{R(\ell)*} \Big) \\ & + \frac{1}{4} I_{4(A,B,X,Y)} m_{\tilde{\chi}_A^0} m_{\tilde{\chi}_B^0} \left(N_{dAX}^{L(d)} N_{sBX}^{R(d)*} N_{\mu BY}^{L(\ell)} N_{eAY}^{R(\ell)*} \right. \\ & + \lambda N_{dAX}^{L(d)} N_{sBX}^{L(d)*} N_{\mu BY}^{L(\ell)} N_{eAY}^{L(\ell)*} \Big), \quad (43) \end{aligned}$$

$$K_R^{n(2)} = -K_L^{n(2)}|_{L \leftrightarrow R}. \quad (44)$$

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